Bubbling Fluidized Bed Scaling Laws: Evaluation at Large Scales

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Scaling laws for bubbling fluidized beds were first proposed some two decades ago. Although it would appear that the scaling laws for bubbling beds are for the most part successful, generally experimental verification has been limited to small-scale equipment. In this paper we apply the simplified scaling laws to a set of four cold-model bubbling fluidized beds covering a tenfold increase in diameter and perform an experimental verification using an analysis of pressure fluctuations. For pressure fluctuation measurements made at a number of radial and axial locations in all four fluidized beds, an "agreement map" is presented in which the spatial variation in similarity behavior is summarized. Despite a generally good match of results between correctly scaled units, agreement tends to degrade toward the vessel walls and bed surface. At higher gas velocity, pressure fluctuation phenomena observed in the largest bed are not mimicked by smaller units scaled using the simplified criteria. © 2005 American Institute of Chemical Engineers *AIChE J,* 51: 2686–2694, 2005
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Introduction

In this work we evaluate the so-called simplified scaling criteria for bubbling fluidized beds, which are of practical importance because of the minimal restrictions they impose on the physical systems to be matched. The criteria can be represented either by the Glicksman et al.[1] scheme of dimensionless groups (Eq. 1) or by the Horio et al.[2] system of equations (Eqs. 2a and 2b) with the additional requirement of constant solid to gas density ratio (Eq. 2c)

$$\frac{gD}{U^2} \quad \frac{\rho_f}{\rho_s} \quad \frac{U}{U_{mf}} \quad \text{geometry} \quad \Phi \quad \text{particle size distribution} \tag{1}$$

$$U_2 - U_{mf2} = \sqrt{m}\,(U_1 - U_{mf1}) \tag{2a}$$

$$U_{mf2} = \sqrt{m}\,U_{mf1} \tag{2b}$$

$$\frac{\rho_f}{\rho_s} \tag{2c}$$

Following from the equivalence demonstrated by Glicksman[3] both the sets presented above are likewise equivalent. Additionally there is evidence to suggest that the so-called chaotic similarity parameter proposed by van den Bleek and Schouten[4]

$$\frac{KD}{U_{mf}} \tag{3}$$

also demonstrates equivalence to the Glicksman and Horio schemes. Van der Stappen[5] and Schouten et al.[6] made the experimental observation that the measured chaotic similarity group appeared to be matched *automatically* at different scales when the simplified scaling approach was precisely followed.

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Issues in experimental verification

When two fluidized beds are hydrodynamically similar, their dependent hydrodynamic phenomena, when expressed in dimensionless form, will be identical. The objective of experimental verification is to measure and compare dependent hydrodynamic phenomena (such as bubble characteristics, solids particle velocity, and bed turnover time) in fluidized beds constructed according to the scaling laws. Glicksman et al.[7,8] provide a comprehensive overview of the approach to fluidized bed scale-up, generally, as well as a detailed summary of the majority of the scaling law experimental verification work conducted to date.

Unfortunately, many of the previous studies did not include any deliberate experimental mismatch in scaling-law parameters. Such a control experiment is necessary to demonstrate the validity of the test procedure being used to verify similarity and neglecting to do this constitutes a significant oversight. Consequently, the results of such studies must be treated with a certain amount of caution.

Additionally, although the bubbling bed scaling laws should be applicable over a wide range of bed sizes, only a few investigators—all of whom used the *full* set of criteria—evaluated them in beds whose cross section was >600 mm.[9-11] The study of Fitzgerald et al.[11] constitutes the largest bubbling bed in which scaling-law verification work has been undertaken—an 1830-mm^2 combustor—although the authors did not achieve hydrodynamic similarity with the smaller-scale unit to the extent that they considered it statistically significant.

In contrast, the largest bubbling bed used in *simplified* scaling-law verification was the 600-mm-diameter cold model used in the solids mixing studies of Horio et al.[12] It is also likely that wall effects play a significant role in bed hydrodynamics at small scales[13,14] and, without more large-scale data on scaling-law verification, the applicability of the scaling laws to large-scale changes cannot be automatically assumed.

In this work we report on an experimental verification of the simplified scaling laws for bubbling fluidized beds undertaken across a wide range of bed sizes (up to 1.56 m diameter). Evaluation is by a comparison of parameters derived from measurements of pressure fluctuations at several locations in the beds. We also demonstrate that the approach used distinguishes between correctly scaled and misscaled systems.

Experimental

Pressure fluctuations in bubbling fluidization

Because of their relative simplicity and the wealth of information that can be derived from them, pressure measurements and pressure fluctuation analyses have been popular experimental tools for investigations of fluidized bed behavior for quite some time. Many of the previous models proposed to explain these fluctuations agree with the simplified scaling-law approach in terms of both pressure fluctuation frequency and amplitude.[15-19] It is therefore reasonable to expect that the measured characteristics of a pressure fluctuation signal should scale according to the simplified scaling laws. Consequently, an analysis of the pressure fluctuations from a pressure probe in a bubbling fluidized bed in terms of amplitude and frequency (and statistical functions of these quantities) is likely to provide a sound basis for a quantitative hydrodynamic comparison of the simplified scaling laws, which is the approach adopted in this work.

In the present work, a number of scales, operating conditions, and comparisons are involved. Whereas several probability density distributions and amplitude spectra are compared to provide some detailed information about the magnitude and frequencies of the pressure fluctuations, respectively, the majority of the comparisons are carried out using the average absolute deviation as a representation of fluctuation magnitude, and the average cycle frequency as a representation of the fluctuation frequency, in a fashion similar to that of Van der Stappen.[5]

Nondimensionalization of the measured parameters (pressure fluctuation magnitude and frequency) was carried out as follows.

In the small-scale tests the magnitude of pressure measurements was nondimensionalized by

$$P^* = \frac{P}{\rho_b g H_s} \tag{4}$$

With larger-scale tests we used the alternative expression

$$P^* = \frac{PA}{Mg} \tag{5}$$

to remove the influence of errors in measured bulk density and settled bed height on the results. Frequency was nondimensionalized by

$$f^* = f\frac{D}{U_{mf}} \tag{6}$$

Apparatus

Four different diameter bubbling fluidized bed cold models were used in the experimental work. Complete details of the setups are provided in Sanderson.[20] They were all of circular cross section, with bed diameters of 146, 300, 600, and 1560 mm. All of the beds were fluidized with ambient air and were equipped with geometrically similar bubble cap distributors[20] and conical windboxes. Each distributor used 18 bubble caps in a triangular-pitch arrangement. The air supply to the smallest bed was from laboratory compressed air, whereas the three large beds were each supplied with air from Rootes blowers. Air to the 146- and 300-mm-diameter beds was metered through variable-area flowmeters and air to the larger beds was metered by orifice plates. Bed materials were Geldart B silica sands (particle sphericity ≅ 0.86), the details of which are given in Table 1. Particle size distributions were closely matched; details can be found in Sanderson.[20]

Pressure fluctuation data were gathered using horizontally mounted single-ended pressure probes. Horizontal mounting was chosen because, although it causes more disruption to bubble passage than the vertical-stemmed probe, the probes are more easily located and less prone to flexing because of the dynamic forces within the bed. The probes were made single-ended because, that way, the effects of the fast pressure waves resulting from the hydrodynamic phenomena occurring

Table 1. Properties of the Silica Sand Bed Materials Used in the Similarity Experiments

Bed Material	Bed Used (mm)	d_{sv} (μm)	ρ_s (kg/m^3)	U_{mf} (m/s)
A	146	225	2650	0.039
B	300	280	2650	0.058
C	600	330	2650	0.070
D	1560	425	2650	0.127
A*	146	337	2650	0.085
B*	300	398	2650	0.125

throughout the bed would be registered, thus more effectively representing the "global" hydrodynamics in the bed.[21] Also, because the local effects registered by the single probe are likely to be small, the disruptive effect of the probe on the local bed conditions is less likely to influence the measured fluctuations. Probe locations used in this work are shown in Figure 1.

To completely rule out the possibility of differences in probe design affecting signals from one bed and not another, the same probe tube geometry (500 mm long, 2.5 mm ID; selected according to van Ommen et al.[22]), transducer type (Data Instruments XCX series), and data-acquisition system (PCL-818HG card in a desktop PC running Advantech Genie v2.12) were used throughout the study. Alternative mounting arrangements were developed for positioning probe tips flush with the vessel walls and also for extending them to the center of the large fluidized bed.[20] In the experiments nearly 16,000 sample points were recorded for each pressure probe measurement. In anticipation of the signal frequencies being higher in the smaller-scale units, data sampling frequencies were increased at the smaller scales as an attempt to obtain approximately the same number of sample data points per average cycle at all scales. Sampling frequencies were 83, 131, 187, and 258 Hz from the largest to smallest bed, respectively. Initial signal testing and pressure fluctuation experiments were performed to demonstrate that (1) signal-to-noise ratios were adequate; (2) the discretely sampled signal accurately represented the pressure fluctuations, without aliasing; and (3) the amplitude and frequency characteristics of the fluctuations for a given set of operating conditions were invariant and reproducible.[20]

Preliminary small-scale experiments

Some preliminary verification work was conducted at small scale (on the 146- and 300-mm-diameter units) to check that the measurements and analyses were adequate and permitted correct discrimination between known cases of hydrodynamic similarity and nonsimilarity.

The experiments were performed with settled height-to-diameter ratios ≅ 2:1. A single pressure probe was used in each bed, with its tip placed at the radial centerline and inserted at a height of 103 mm above the distributor in the 146-mm-diameter bed and 212 mm above the distributor in the 300-mm-diameter bed. In these initial trials, the effects of operating gas velocity and bed material selection were investigated. Experiments were repeated three times at each operating condition.

Two experiments were performed:

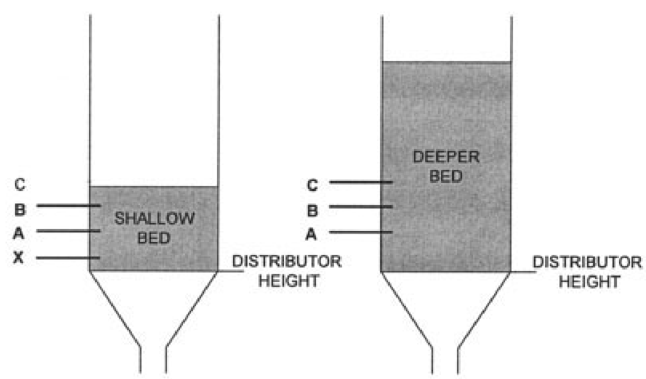
Experiment 1. In the 146-mm bed, material A was used and compared with its correctly scaled counterpart, material B, in the 300-mm bed. This experiment was designed to represent a correctly scaled system.

Experiment 2. In a secondary set of experiments, material A* was used in the 146-mm bed, designed to scale correctly with its counterpart, material B* in the 300-mm system. This experiment was also designed to represent a correctly scaled system, but involved larger particle sizes and thus higher particle Reynolds numbers than those in Experiment 1.

Note that by comparing results of Experiment 1 with Experiment 2 at appropriate operating velocities (that is, the same U/U_{mf}), the consequences of a misscaled particle size can be determined. (The properties of materials A, B, A*, and B* are defined in Table 1.)

For systems that followed the simplified similarity criteria, good agreement in all the pressure fluctuation characteristics was observed. For examples, refer to Figures 1 and 2, which respectively compare the average absolute deviation of dimensionless pressure and dimensionless average cycle frequency vs. dimensionless superficial gas velocity for correctly scaled and misscaled cases. This is consistent with the observations from the small-scale tests of other investigators.[5,19,23-25] These preliminary experiments also demonstrated that the disagreement in trends for the deliberately misscaled particle sizes is pronounced. The comparison shows how the misscaling is reflected in the experimental measurements and demonstrates not only the mismatch associated with not following the scaling laws, but also that the measurements used are able to correctly distinguish between scaled and misscaled systems.

Figure 3 shows the normalized probability density distributions for correctly scaled case of materials A and B, indicating that the distribution of the dimensionless pressure fluctuations from the two units are indeed very similar, supporting the more



Bed:	X	A	B	C
146 mm	19.5	48	75	103
300 mm	40	96	154	212
600 mm	80	192	308	424
1560 mm	210	500	800	1100

Figure 1. Vertical distance from top surface of distributor plate to each pressure tapping point.

The tapping point heights correspond to the same dimensionless probe height (h/H_s) at each scale.

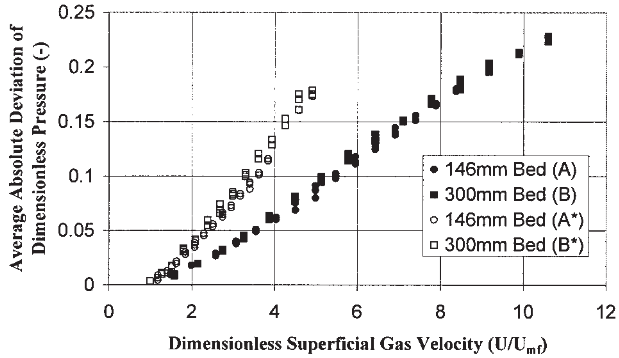


Figure 2. Results for the average absolute deviation of dimensionless pressure for correct and misscaled beds.

Materials A and B in the 146- and 300-mm beds, respectively, are correctly scaled. Materials A* and B* in the 146- and 300-mm beds, respectively, are also correctly scaled, but different from the A–B pair.

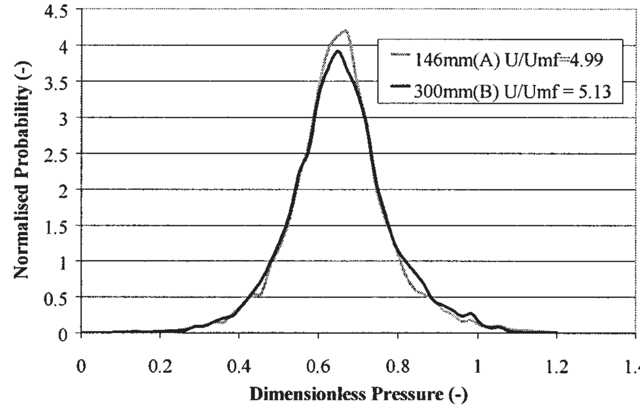


Figure 4. Comparison of the normalized probability distribution for dimensionless pressure fluctuations from the 146-mm bed with material A and the correctly scaled 300-mm bed with material B at intermediate gas velocity.

generalized result from the comparison of the average absolute deviation of the pressure fluctuations (Figure 1).

In Figure 4, sample comparisons of selected frequency spectra are presented for the correctly scaled case of materials A* and B*. The agreement in the shape and size of the amplitude spectra presented in the figure is indicative of closely matching behavior in the scaled beds. In comparing additional spectra (not shown) we found that similar to the results for average cycle frequency (Figure 2), the spectra show only a negligible dependency on gas velocity. For comparison, spectral results from a particle size mismatch are presented in Figure 5, clearly showing a significant difference. These results demonstrate consistency between the simple dimensionless average cycle frequency and spectral representations of pressure data, showing that the former approach can be confidently used to demonstrate similarity between scaled systems and to correctly distinguish between scaled and misscaled systems.

A further use of the preliminary experiments is that the scatter from repeating experiments enabled estimation of random error, which was found to be not so significant as to prevent the scaled and misscaled systems from being correctly distinguished.

The results at the small scales present a benchmark from which to evaluate further similarity comparisons for larger changes in bed dimensions.

Large-scale experiments

The large-scale comparison of pressure fluctuations was carried out for all four fluidized beds, of diameters 146, 300, 600, and 1560 mm. As a result of blower limitations in the 1560-mm bed, all four beds were operated with settled aspect ratios of 2:3 (H_s:D). Silica sands, with properties outlined in Table 1, were fluidized under the conditions shown in Table 2. In all, there were five fluidized bed setups explored: all four bed sizes were first scaled according to the simplified scaling laws and an additional misscaled run was then carried out

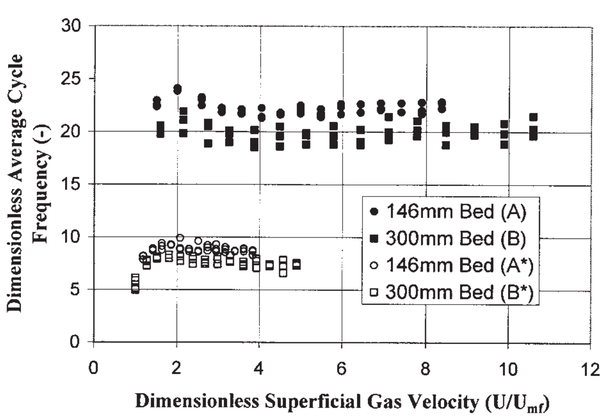


Figure 3. Comparison of the dimensionless average cycle frequency for the pressure fluctuation data for all preliminary experiments.

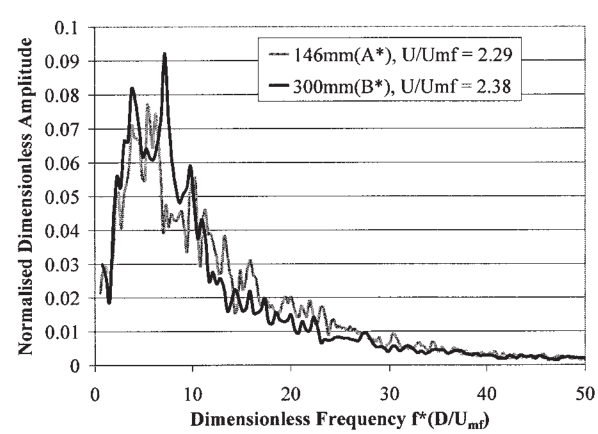


Figure 5. Comparison of the normalized ensemble-averaged amplitude spectra for the dimensionless pressure fluctuations from the 146-mm bed with material A* and the correctly scaled 300-mm bed with material B* at intermediate gas velocity.

Table 2. Ranges of Superficial and Dimensionless Superficial Gas Velocities and Particle Reynolds Number for the Hydrodynamic Similarity Experiments*

Material	Bed (mm)	U Range (m/s)	U/U_{mf} Range	Re_p Range
		2:1 (H_s/D) Aspect Ratio Experiments		
A	146	0.058–0.327	1.49–8.37	0.9–6.4
B	300	0.092–0.614	1.58–10.59	1.8–11.7
A*	146	0.101–0.327	1.18–3.84	2.4–9.5
B*	300	0.159–0.614	1.51–4.91	4.1–15.9
		0.67:1 (H_s/D) Aspect Ratio Experiments		
A	146	0.042–0.146	1.06–3.75	0.7–2.3
B	300	0.062–0.217	1.06–3.75	1.2–4.1
A (higher U)	146	0.048–0.362	1.23–9.28	0.8–5.7
B (higher U)	300	0.071–0.539	1.22–9.29	1.4–10.3
C	600	0.082–0.660	1.17–9.43	1.8–14.8
D	1560	0.173–0.515	1.36–4.06	5.3–15.7
D	600	0.141–0.646	1.11–5.09	4.6–21.2

*The particle Reynolds number (Re_p) is based on fluid density at exit conditions and surface volume mean diameter. "Higher U" refers to separate runs conducted at higher gas velocity.

where material from the 1560-mm bed was fluidized in the 600-mm bed (particle size mismatch).

Pressure probes were installed at three axial locations in each bed corresponding to the same dimensionless height (see Figure 6). Three radial positions for the probe tips were chosen: (1) at the bed wall ($r/R = 1$), (2) halfway between the vessel wall and the bed center ($r/R = 0.5$), and (3) at the bed center ($r/R = 0$). This meant that pressure fluctuations were recorded from a total of nine probe locations in each bed over the course of the experiments. Similar to procedures of the small-scale work, experiments were repeated three times to give some indication of the reproducibility of the results (except in the case of the misscaled run in the 600-mm bed, carried out only once for each probe location).

The analysis of the pressure fluctuations followed lines similar to those established in the small-scale work; the majority of the comparisons were made by the dimensionless average absolute deviation and the average cycle frequency of fluctuations registered from the individual probes within each of the beds.

A comparison of the average absolute deviation results for

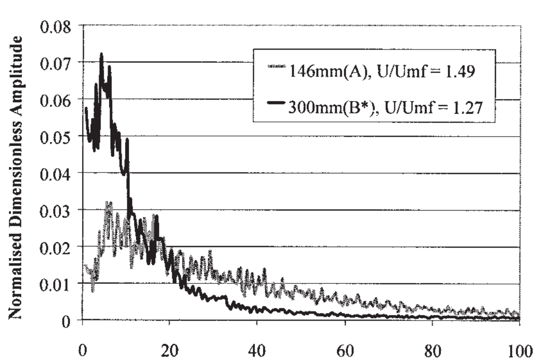


Figure 6. Comparison of the normalized ensemble-averaged amplitude spectra for the dimensionless pressure fluctuations from the 146-mm bed with material A and the 300-mm bed with mismatched bed material B* at low gas velocity.

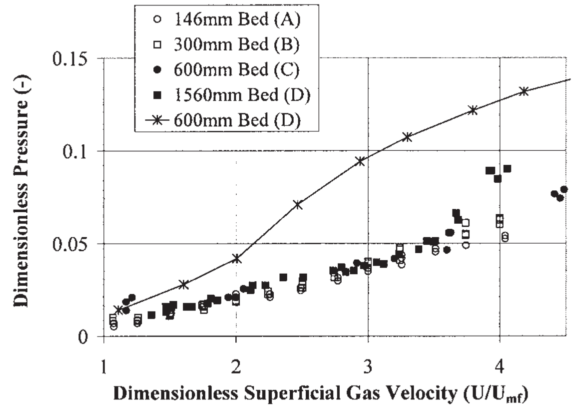


Figure 7. Comparison of the dimensionless average absolute deviation of pressure measured from pressure probes located at $h/H_s = 0.77$ and $r/R = 0$ in all five fluidized beds for a range of dimensionless gas velocities.

All beds, with the exception of the 600-mm bed with material D, have been scaled using the simplified scaling criteria.

the probe positioned near the bed surface and at the bed center is presented in Figure 7. These results are typical of those measured at all probe locations. The trend for the misscaled bed is dramatically different—as expected from the results of the small-scale work—clearly indicating the increase in fluctuation amplitudes associated with the larger-than-scaled bubbles present in this bed. Agreement between the other four beds is good, with the exception of the 1560-mm bed results at the highest gas velocities, where the average absolute deviation of the pressure signals increases rather dramatically compared to its correctly scaled counterparts. Additionally, at the same gas velocity, it was also noted that the dimensionless average bed pressure (not shown) in this bed started to decrease (when compared with the correctly scaled smaller models).

A typical comparison of average cycle frequency results for the five beds is shown in Figure 8. The agreement is reasonable between correctly scaled beds, and there is a clear distinction

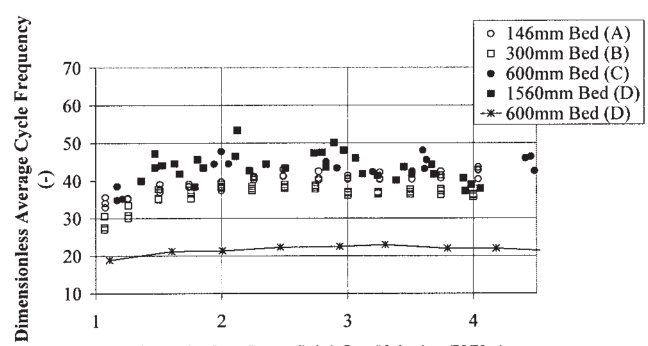


Figure 8. Comparison of the dimensionless average cycle frequency of pressure measured from pressure probes located at $h/H_s = 0.46$ and $r/R = 0$ in all five fluidized beds for a range of dimensionless gas velocities.

All beds, with the exception of the 600-mm bed with material D, have been scaled using the simplified scaling criteria.

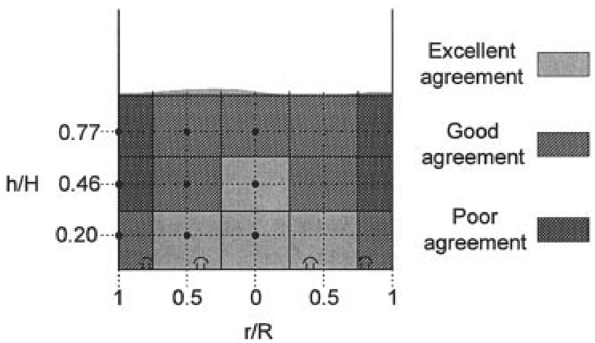


Figure 9. Agreement map showing qualitatively how well the pressure fluctuations from the various probe locations and superficial gas velocities from 1.25 to 3.5U_{mf} match for the scaled fluidized beds.

Black dots indicate the location of the probe tips in the actual measurement runs; the results have been extended across the bed width assuming the behavior to be axisymmetric (excellent agreement = trends are indistinguishable; good agreement = trends are similar with some scatter; poor agreement = trends are only marginally better than for the misscaled scenario).

between the correct and misscaled scenarios. Generally speaking, frequency results were slightly more scattered for the larger beds (1560 and 600 mm) than for the smaller beds, and the scatter was most noticeable at low gas velocities for probe positions toward the bed surface. There was no discernible change in the average cycle frequency trend for the largest bed at gas velocities where the previously mentioned differences in bed pressure drop average and average absolute deviation were noticed. The dimensionless average cycle frequency in the correctly scaled 600-mm bed appears somewhat higher than that of its smaller counterparts, which is a consequence of the slightly larger error in the minimum fluidization velocity for this bed material. (A correct match requires $U_{mf} \approx 0.08$ m/s and this bed material has $U_{mf} = 0.07$ m/s; thus when nondimensionalizing the measured frequency, a slightly higher dimensionless value is obtained.)

To summarize the results with respect to the success of the scaling criteria, an "agreement map" has been generated (Figure 9), which indicates qualitatively how well the pressure fluctuations agree (in terms of average bed pressure drop, average absolute deviation, and average cycle frequency) for the four scaled beds. The map is based only on a qualitative comparison of the results and is designed to show how the agreement varies with probe location. The diagram is drawn assuming axisymmetric behavior, thus extending the results across the full bed width. Results are mapped for the scaled beds at velocities up to 3.5U_{mf}, to exclude the change in behavior observed in the 1560-mm bed at high velocity, which will be addressed separately. Overall, the agreement between the correctly scaled beds is good. Results near the distributor and the bed center compare well; however, it should be noted that the agreement becomes worse toward the bed surface and wall.

In Figure 10, the probability density function results for the 300- and 1560-mm beds are compared with the mismatched 600-mm bed results, to demonstrate the significant difference between the distributions for a correctly matched and a mismatched bed pair. Although the distributions are centered approximately around the same average value, the distribution for the mismatched bed is much wider, indicating a much wider range of dimensionless pressure fluctuation amplitudes is present. These trends were also observed for other probe locations.

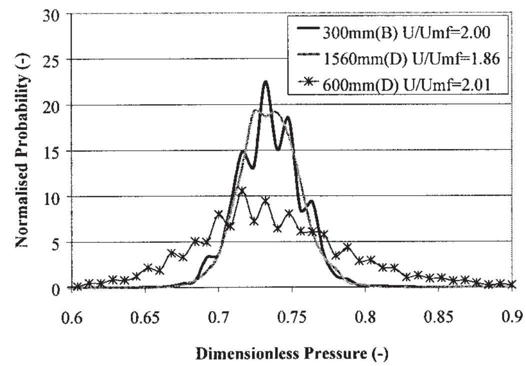


Figure 10. Comparison of the normalized probability distributions for the correctly scaled beds (300 mm, material B; 1560 mm, material D) with the mismatched bed (600 mm, material D) at low gas velocity for the probe located at $r/R = 0$ and $h/H = 0.2$.

We now examine the probability density function results for the region of high-velocity operation ($U/U_{mf} = 4$), where the 1560-mm bed showed an increase in average absolute deviation and a decrease in average pressure drop. Figure 11 shows the comparison of the probability density function for the correctly scaled 146-, 300-, and 1560-mm beds. These results clearly show the large spread of the pressure fluctuation amplitudes and the reduction in the average bed pressure drop for the 1560-mm bed that is not reflected in its smaller-scale counterparts at this value of U/U_{mf}.

For clarity, a comparison of typical amplitude spectra results (from the probe located at $r/R = 0$ and $h/H_s = 0.2$) for correctly scaled beds is presented in three figures (Figures 12–14). The 146-mm bed is taken as the reference. For the three smaller beds agreement in terms of spectral content of the pressure

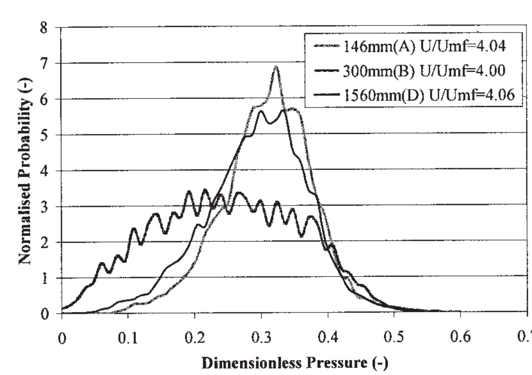


Figure 11. Comparison of the normalized probability distributions for the correctly scaled beds (146 mm, material A; 300 mm, material B; 1560 mm, material D) at high gas velocity for the probe located at $r/R = 0$ and $h/H = 0.77$.

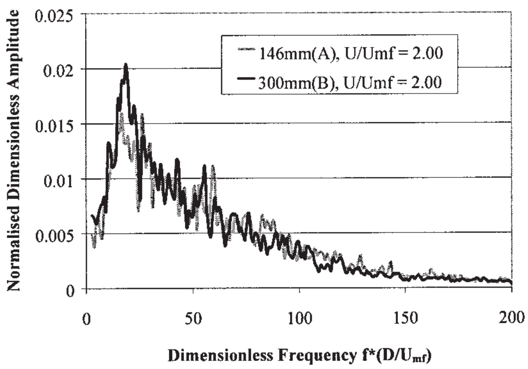


Figure 12. Comparison of the dimensionless amplitude spectra from the correctly scaled 146- and 300-mm beds at low gas velocity, with probe located at *r/R* = 0 and *h/H* = 0.2.

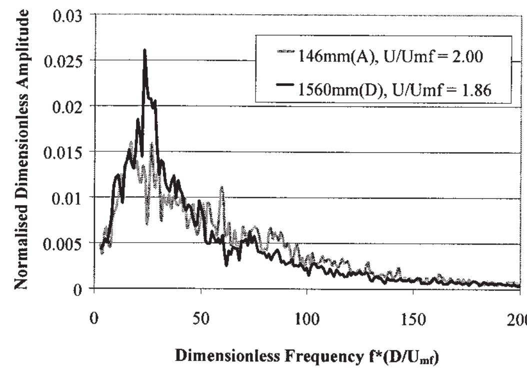


Figure 14. Comparison of the dimensionless amplitude spectra from the correctly scaled 146- and 1560-mm beds at low gas velocity, with probe located at *r/R* = 0 and *h/H* = 0.2.

signals appears to be reasonable. Figure 14 shows that there are some differences between the spectrum of the 1560-mm bed and that of the 146-mm bed (these were also indicated by the average cycle frequency results of Figure 8). For comparison, the results for the deliberately misscaled 600-mm bed are shown with data from the 146-mm bed in Figure 15. Comparatively, the 1560-mm bed shows a somewhat better agreement than that for the misscaled 600-mm bed, yet not as good as the agreement between the correctly scaled smaller beds.

Discussion

From the experimental results, there are two areas of particular interest: (1) the spatial variations in hydrodynamic similarity summarized in the agreement map (Figure 9); and (2) the different fluctuation results trend for the largest bed at high gas velocity (above $U/U_{mf} = 3.5$). These will now be addressed in turn.

Spatial variations in hydrodynamic similarity

As the qualitative agreement map of Figure 9 shows, the best agreement across the full range of bed diameters was obtained (for gas velocities $< 3.5U_{mf}$) for pressure probes located away from the walls and the bed surface. Agreement is generally good near the distributor, which tends to indicate that the initial formation of bubbles occurs in a hydrodynamically similar way; however, it seems likely that wall effects and surface effects cause some of the differences noted in the various beds at lower gas velocities.

Of the few previous verification studies that measured and compared pressure fluctuation results from different probe locations, most found some form of dependency on probe location. Contrary to the present work, van der Stappen[5] found the agreement from single pressure probes in beds scaled by the simplified scaling laws to be particularly poor at the distributor and better further up the bed. The difference in this case was attributed to the effect of the distributors, which were not explicitly scaled and may have caused misscaled behavior in the initial bubble formation region of the bed. Almstedt and Zakkay[10] used capacitance probes, although the results in terms of bubble behavior are relevant here. They used the full set of scaling laws and also found poor agreement near the distributor. Once again, however, differences were attributed to a known difference between distributors in the two beds (that is, that a significant temperature gradient across the distributor in the hot model was abruptly changing the gas transport properties—an effect that could not be replicated in an ambient

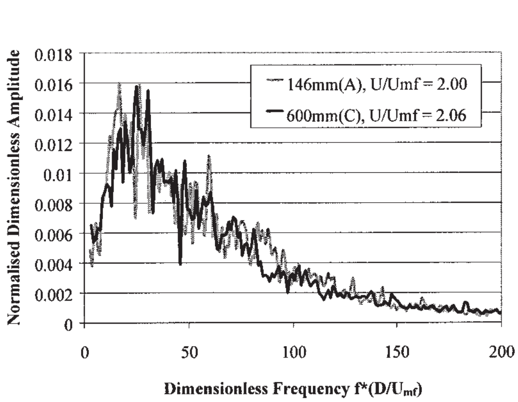


Figure 13. Comparison of the dimensionless amplitude spectra from the correctly scaled 146- and 600-mm beds at low gas velocity, with probe located at *r/R* = 0 and *h/H* = 0.2.

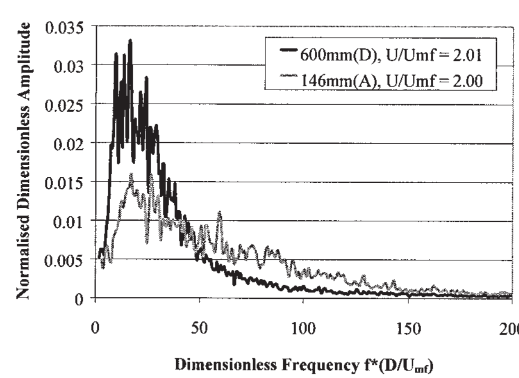


Figure 15. Comparison of the dimensionless amplitude spectra from the correctly scaled 146-mm and the misscaled 600-mm bed at low gas velocity, with probe located at *r/R* = 0 and *h/H* = 0.2.

model). Nicastro and Glicksman[9] evaluated the full set of scaling laws and used differential pressure probes. They found generally good agreement, although agreement of the results was slightly poorer toward the lateral center of the bed than at the walls. The difference, however, was only slight. Finally, Brue and Brown[26] used differential pressure measurements from wall tappings in beds scaled using the full set of scaling criteria and found quite clearly that the closer the measurements were to the bed surface, the poorer the agreement of the results. They attributed the differences to bursting bubbles or sloshing of the emulsion phase—phenomena not explicitly accounted for by the scaling criteria. So, whereas in the "ideal world" of complete hydrodynamic similarity the effect of the wall and bed surface on bed behavior would be scaled exactly with bed dimension, the experimental results of previous studies show that it may not be the case, and that improper distributor matching may also cause trouble.

Van Ommen et al.[27] applied the attractor comparison method proposed in van Ommen et al.,[28] as an alternative means of interpreting the results from some of the experiments presented here. For measurements made at the wall of the 146-, 300-, and 1560-mm beds they also confirm the same trend for decreasing similarity toward the bed surface, although the technique suggests a far more dramatic lack of similarity than do the analysis methods presented here. It remains to be seen whether their requirements for similarity are too strict for this application, although the technique is rigorous and avoids the subjective interpretation of results based on a visual comparison of trends. The reader is referred to van Ommen et al.[27] for further details of this alternative analysis.

Changes in bed behavior with changing gas velocity

As mentioned earlier, for gas velocities exceeding around $3.5U_{mf}$, the average pressure begins to drop and the amplitude of the pressure fluctuations begins to rise dramatically. Several possible options for the underlying phenomenon causing the change were considered.

The possibility that it was related to a limiting particle Reynolds number (Re_p) above which the simplified criteria do not apply can be dismissed, given that subsequent experiments in the smaller beds at higher gas velocities achieving similar particle Reynolds numbers failed to produce a similar effect.[20] Other workers[5,19,23-25] also presented results indicating success with the simplified scaling criteria with one or more scaled beds operating at $Re_p > 13$, the value at which the observed discrepancy commences in this work.

None of the typical bubble size correlations[29-32] predicts bubble size changes that would cause the observed difference in 1560-mm-bed behavior at higher gas velocities. Likewise, according to the various minimum slugging criteria,[30,33-35] none of the beds is deep enough for slugging to occur.

To the best of the authors' knowledge, there is no single obvious explanation for the discrepancy in the 1560-mm-bed behavior that occurs at gas velocities beyond $U/U_{mf} = 3.5$. The lowering of average pressure measured by the probes toward the radial center tends to indicate the probes are encountering an increased voidage between their location and the bed surface. The increase in amplitude also implies increasingly violent fluctuations in bed height. The change in behavior is attributed to a phenomenon not accounted for in the simplified scaling laws; it is not predicted by bubble size correlations; it is not explained by a transition to slugging; and the combination of high-pressure drop distributor and relatively low gas velocities used suggest that the single bubble or exploding bubble regime has not been reached. Despite the lack of *prediction* for this behavior, results from the pressure fluctuation measurements clearly indicate that a change does indeed occur in this large bed and, because the underlying cause of this change is unclear, it serves as grounds for future work.

Conclusions

Based on the verification literature for the simplified scaling laws for bubbling fluidized beds, we identified that there was a lack of verification data at large scales, and that only a limited number of previous studies included deliberate mismatch experiments to confirm the validity of their experimental techniques.

In the present work we have attempted to address these points. We have extensively evaluated the simplified scaling laws for bubbling fluidized beds using pressure fluctuations measured from a number of bubbling fluidized beds covering a tenfold increase in bed diameter. Initially we verified the validity of our measurement technique by evaluating scales and conditions similar to those used by previous workers, incorporating both correctly matched and misscaled experiments. At smaller scales, the simplified scaling laws were found to be generally successful for the majority of conditions studied.

Upon extending the evaluation to encompass bed diameters up to 1560 mm we found that for gas velocities up to $3.5U_{mf}$, the agreement in bed behavior was best toward the radial center and in the lower portions of the bed. There was an identifiable degradation in the observed similarity close to the wall and bed surface.

At gas velocities $> 3.5U_{mf}$, the largest bed (1560 mm diameter) showed a distinct deviation in average pressure drop and fluctuation amplitude that was not mimicked in the correctly scaled smaller units, even at substantially higher gas velocities. The observed phenomenon could not be accounted for by the scaling criteria, bubble size predictions, slugging criteria, bubble regime change, or a limiting particle Reynolds number.

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Notation

A = bed cross-sectional area, m^2
D = bed diameter, m (unless otherwise indicated)
d_p = mean particle diameter, μm
d_{sv} = Sauter mean particle diameter, μm
f = measured frequency, Hz
f^* = dimensionless frequency
g = acceleration arising from gravity, m/s^2
H_s = settled bed height, m
h = axial distance from distributor, m
K = Kolmogorov entropy, bits/s
M = bed mass, kg
m = scaling length ratio
P = measured pressure, Pa

P^* = dimensionless pressure
R = bed radius, m
r = radial distance from bed center, m
U = superficial gas velocity, m/s
U_{mf} = minimum fluidization velocity, m/s
μ = gas viscosity, kg/ms
ρ_b = solids bulk density, kg/m^3
ρ_f = fluid density, kg/m^3
ρ_s = particle density, kg/m^3
Φ = particle sphericity
1, 2 = original and scaled bed properties, respectively
Re_p = particle Reynolds number = $\rho_f U d_{sv}/\mu$

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